Little hierarchy in the minimally specified MSSM

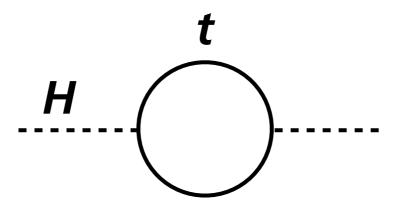
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Fine tuning problem in EWSB

in a bottom up approach is seen as the cancellation of two numbers:

the top loop contribution to the Higgs mass squared



$$\Delta \mathbf{m_H^2} = -\frac{\lambda_t^2}{8\pi^2} \Lambda^2 + \dots$$

and the boundary condition at the scale of new physics.

Quantifying fine tuning

In a model with two parameters A, B \sim 1 contributing to X,

$$X = A - B$$

in order to get e.g. X = 0.001, parameters A and B have to be specified with 3 digits and carefully chosen/tuned

e.g.: A = 0.963

B = 0.962

Tuning often quantified by:

$$\max_{p=\{A,B\}} \left| \frac{\partial \ln X}{\partial \ln p} \right| \simeq \frac{A}{X} \simeq 1000$$

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Note that ~10% tuning corresponds to specifying A and B with one digit: no matter what the remaining digits are, we need to adjust just the first digit of model parameters to get X ~ 0.1

Fine tuning in the MSSM

In the MSSM there are several parameters contributing to the electroweak scale, e.g:

$$M_Z^2 \simeq -1.9\mu^2 + 5.9M_3^2 - 1.2m_{H_u}^2 + 1.5m_{\tilde{t}}^2 - 0.8A_tM_3 + 0.2A_t^2 + \dots$$

boundary conditions at the GUT scale $\tan \beta = 10$

The usual naturalness measure,

$$\max_{p} \left| \frac{\partial \ln M_Z^2}{\partial \ln p} \right|$$

only cares about the largest individual contribution. It doesn't tell us how the model parameters need to be tuned/specified.

Little hierarchy from complexity

e.g. in a model with more parameters contributing to X:

$$X = A - B - c - d$$

in order to get X = 0.001 for randomly chosen $A \sim 1$, no parameter needs to be carefully chosen, e.g.:

$$A = 0.963$$
 $B = 0.9$ $c = 0.06$ $d = 0.002$

only the first digit of all parameters need to be adjusted no matter what the following digits are

what is a tuned outcome in a model with 2 parameters may be a completely ordinary outcome in a more complex model

RD, arXiv:1611.03188

Outline

Naturalness criteria based on the largest contribution are too strong and do not necessarily indicate how model parameters have to be tuned.

Naturalness requirement (in this talk):

Natural outcome for an observable in a given model is any outcome that does not require specifying more than one digit of model parameters, regardless of what the remaining digits are.

Let's see what it means for the CMSSM.

Assume model parameters of the same order

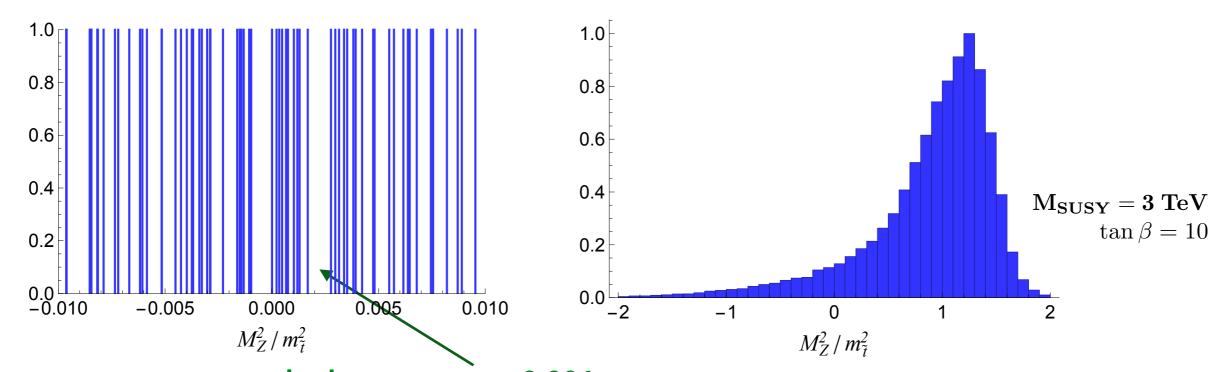
$$\mathbf{M_{1/2}},\ \mathbf{m_0},\ \mu,\ -\mathbf{A_0}\ \simeq\ \mathbf{M_{SUSY}}$$

and vary them in \pm 50% range keeping only one digit specifying the departure from $M_{\rm SUSY}$, e.g.:

$$\begin{array}{lll} M_{1/2} = 0.6\,M_{SUSY} & M_{1/2} = 0.9\,M_{SUSY} & M_{1/2} = 0.5\,M_{SUSY} \\ m_0 = 1.1\,M_{SUSY} & m_0 = 1.2\,M_{SUSY} & m_0 = 1.1\,M_{SUSY} \\ \mu = 1.4\,M_{SUSY} & \mu = 0.6\,M_{SUSY} & \mu = 1.5\,M_{SUSY} \\ -A_0 = 0.9\,M_{SUSY} & -A_0 = 0.8\,M_{SUSY} & -A_0 = 0.8\,M_{SUSY} \end{array}$$

Maximal hierarchy from minimally specified inputs:

RD and N. McGinnis, arXiv:1705.01910

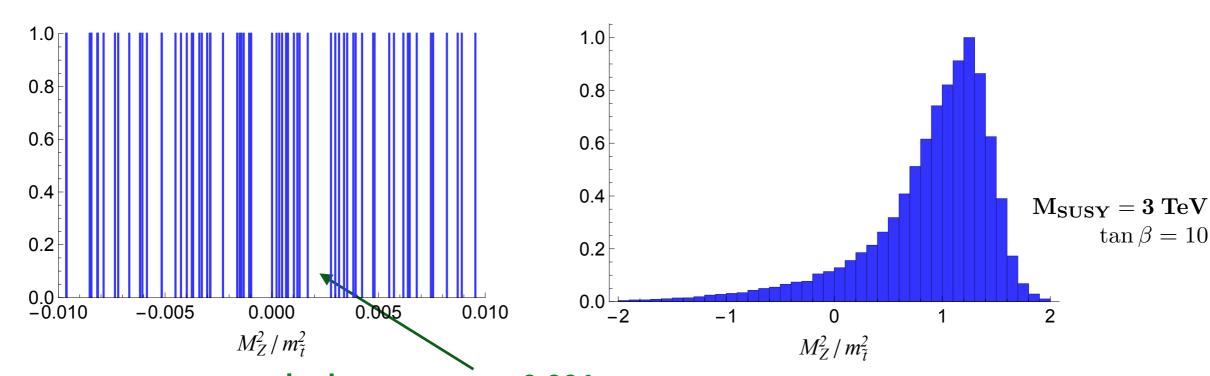


the largest gaps ~0.001 (fairly uniform away from edges of the distribution)

The smallest outcome that does not depend on specifying parameters with more than 1 digit is indicated by the largest gap found in the distribution. Outcomes smaller than the largest gap are accidental.

Maximal hierarchy from minimally specified inputs:

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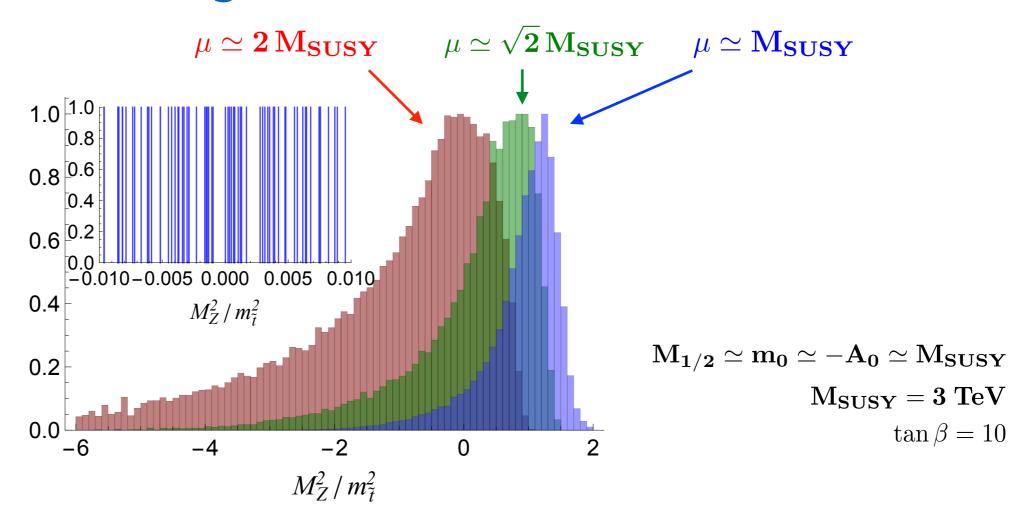
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The smallest outcome that does not depend on specifying parameters with more than 1 digit is indicated by the largest gap found in the distribution. Outcomes smaller than the largest gap are accidental.

$$m m_{ ilde{t}} \simeq 30 \, M_{Z}$$

is an ordinary outcome from minimally specified parameters

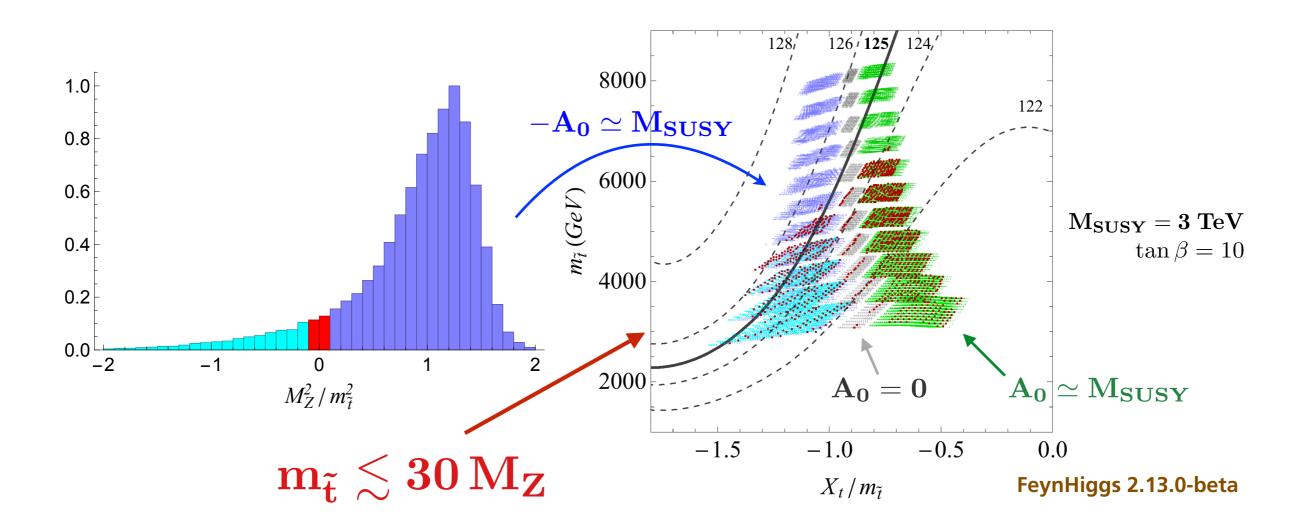
For different central values of parameters, the peak of the distribution changes



but the maximal gap size remains almost the same.

Prediction for maximal hierarchy is very robust.

Higgs boson mass from minimally specified inputs:



Only scenarios with negative A-terms can have sufficiently heavy stops to explain the Higgs boson mass

Conclusions

The usual naturalness criteria do not necessarily indicate how model parameters have to be tuned.

I advocated considering any outcome that does not require specifying more than one digit of model parameters as natural.

In the CMSSM, up to ~3 TeV superpartners are natural in this sense. Only one digit of model parameters needs to be adjusted to get ~100 GeV electroweak scale, no matter what the remaining digits are. Parameter choices with negative A terms can give the correct Higgs mass.